

A REVIEW OF CUMULATIVE DAMAGE  
FOR FATIGUE COMMITTEE  
OF THE STRUCTURES AND MATERIALS PANEL  
ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

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The analysis of any practical fatigue problem almost invariably involves some consideration of the progressive accumulation of damage due to loads that vary in amplitude. It is the purpose of this paper to review the current state of the art of such analysis and to make recommendations for future action.

Advance copies of the review were submitted to other members of the Fatigue Committee and their associates for comment. The writer gratefully acknowledges their contributions and several of these have been included in the present version of the report.

*Author*

A bibliography of papers on cumulative damage published by NASA, NLR, DVL, and LBF is appended.

#### HISTORICAL BACKGROUND

The majority of fatigue tests, both constant and variable amplitude, that have been performed during the many years that fatigue has been studied have been of the rotating-beam type. In a large percentage of these investigations, carefully polished smooth specimens were employed.

Some 40 years ago, Palmgren (ref. 1) proposed a form of cumulative damage relationship that ultimately led to what is now referred to as the Miner (sometimes Palmgren-Miner) relationship (ref. 2). This relation is based upon the simple concept that "damage" accumulates linearly with cycle-ratio and failure occurs when the accumulated damage or sum of cycle ratios reaches unit ( $\sum \frac{n}{N} = 1$ ).

The linear rule of cumulative damage has been used extensively in spite of the fact that a large number of investigations have "proved it wrong". Early evidence that  $\sum \frac{n}{N} \neq 1$  was gained in rotating-beam tests employing simple two-level load schedules. Generally, investigations found that if the stress level was changed from a high to a lower value the summation of cycle ratios was less than unity. Conversely, a sum greater than one was frequently observed when the stress level was increased during a test. These observations led to a great deal of speculation regarding the effects of overstressing, understressing, and coaxing.

Very probably some of the apparent inconsistency in the value of  $\sum \frac{n}{N}$  achieved in a given set of tests is due to inaccuracies in the S-N curves used to perform the calculations.

Some of the investigators proposed relations of their own (refs. 3-16). Generally, these relations involve certain linear or non-linear weighting factors on the cycle ratios employed by Palmgren and Miner. A few have the added feature that the S-N curve is progressively modified to account for the systematic depletion of life and of the fatigue strength remaining in the specimen. All are essentially empirical relations. Some of the authors have claimed superiority of their procedures over the linear method, but this superiority is usually achieved by adjusting empirical constants to suit the data under consideration. Few of the relations proposed attempt to account for the actual mechanisms of damage.

An objective comparison between test results taken from the literature and predictions made by the various cumulative damage methods has been made by the Lockheed Aircraft Company (ref. 17). These investigators concluded that the linear cumulative damage theory gave estimates of fatigue life that were more nearly correct in more cases than were estimates made by any other method evaluated. No trend was evident that any estimation procedure was superior to the linear method for any class of test results.

Thus, the linear method appears attractive because of its simplicity and general superiority, at least for preliminary estimates. Significant discrepancies between observed life and predicted life are, of course, common.

## CURRENT STUDIES

### Parameters in Experimental Studies

More recently, several new parameters have been introduced in order to more adequately simulate service or design problems. Among these are:

1. More complex loading schedules have been used since they were made possible by the development of automatic programming equipment.
2. Axially-loaded specimens were employed.
3. Mean stresses, which could not be applied to rotating beams, became possible and were employed.
4. Specimens frequently incorporate structural joints or at least moderate stress concentrations.
5. A significant number of "research" fatigue tests have been conducted on full-scale structures in Australia, England, the United States and the Netherlands.

In practice, more and more ad hoc fatigue tests are being conducted with programs of loads simulating the expected service experience.

### Phenomenological Trends

Among the systematic trends observed in many of these more recent tests are the following:

1. The fact that the linear summation of cycle ratios does not always equal one is rather universally accepted as a fact. However, the procedure is widely used as a tool for making estimates in design.

2. Systematic variations in  $\sum \frac{n}{N}$  due to various parameters of test programs are being recognized and cataloged. For example:

a.  $\sum \frac{n}{N}$  tends to be greater than one for tests in which a tensile mean stress is used, and increases with the mean stress. This observation suggests that  $\sum \frac{n}{N} = 1$  yields estimates on the "safe side" for many practical problems. (Dr. Gassner advises that he and Dr. Jacoby have some unpublished data which show that  $\sum \frac{n}{N}$  may be in error on the unsafe side by large margins even in cases where positive mean stresses were employed.)

b.  $\sum \frac{n}{N}$  tends to be one or less for tests in which stresses are completely reversed.

c. The range of  $\sum \frac{n}{N}$  values is smaller for complex loading schedules than was observed in the earlier overstressing and coxing tests.

3. The sequence of loading employed in block tests may have a significant effect on fatigue life obtained. This observation dictates that caution be used in setting up fatigue tests for establishing the safe life of flight vehicles.

4. Tests conducted under a random sequence of loading frequently produce lives shorter than those observed in block tests for the same frequency distribution of loads.

5. Ground-air-ground and other negative load cycles can have a much more severe effect on life than was previously recognized or predicted from theories. Reductions in life by factors of 2 to 4 compared to similar tests without negative loads are common. The linear theory might predict only a few percent reduction in some of these cases.

6. High positive load cycles can improve resistance to failure under subsequent lower loads. This effect may be observed even after a crack is present in the structure.

7. On the other hand, once a crack is present, a sufficiently high load can produce catastrophic failure.

#### Significance of Crack Propagation

The role of cracks in the fatigue failure process is getting much more attention than heretofore. The presence of such cracks and their extent is the only measure of the total damage that has been accrued by a specimen or structural component. The rate at which such a crack propagates and the residual static strength for any crack length are the only reliable indices of the remaining longevity of the component in question.

In practical situations, visible fatigue cracks may be present for one-half or more of the total fatigue life. The fact that some microscopic crack growth has occurred before the crack becomes visible suggests that sub-microscopic cracks may be present in many parts before the parts are placed in service.

These considerations are being employed to develop rational methods for estimating life under variable-amplitude loading. The first analytical procedure proposed along these lines is that of Valluri (ref. 16). Valluri developed his expression from basic considerations of local plasticity, then through crack initiation, crack propagation and finally to failure. While this is a rational approach, its quantitative usefulness is subject to question because many of the observations listed earlier cannot be accounted for in the present form.

#### Significance of Residual Stresses

Many of the observations regarding the increase of  $\sum \frac{n}{N}$  with mean stress, beneficial effect of high loads, deleterious effect of negative loads and sequence effects are being rationalized on the basis of residual stresses formed when the material near a stress raiser is plastically deformed. Fundamental studies are underway that will help to define these residual stresses and how they modify fatigue behavior (ref. 18).

To date, only the Smith method (ref. 7) attempts to account for the effects of residual stresses on cumulative damage. Although this method has a basically sound foundation, the assumptions made in the quantitative solution of a given problem appear primitive. It is quite possible that as additional detailed information regarding local plastic stresses becomes available, a method can be developed that will inspire more confidence.

#### Interpretation of Random Loadings

The interpretation of random time histories of loadings in terms of the fatigue damage that they produce is currently receiving increased attention. A wide variety of "counting schemes" by which time histories may be analyzed have been proposed (ref. 19). Unfortunately, analytical schemes for computing the statistics of the necessary quantities are not yet available. Power

spectral density procedures are widely acclaimed for treating random processes, but cannot be employed without a reliable quantitative theory of cumulative damage nor can they provide information on ranges of stress excursions and other parameters generally accepted as controlling fatigue behavior.

## THE OUTLOOK FOR THE FUTURE

Future developments to improve the state of the art with respect to cumulative damage will probably include increased emphasis on the mechanisms involved, namely, crack propagation and failure at a level lower than the original strength of a component. Since fatigue in service is experienced as a result of loads that vary in a random sequence, more emphasis will be placed on tests that employ complex load histories. The systematic effects of residual stresses produced by large positive and negative loads will need to be taken into account, at least in principle.

It is most unlikely that a cumulative damage theory will be developed in the foreseeable future which will eliminate the need for considerable ad hoc testing to establish the adequacy of particular pieces of hardware. The most significant feature controlling fatigue behavior is, of course, the "quality" of the part itself. A truly formidable array of interacting parameters controls "quality". Among these are the material, its heat treatment and other processing details, the configuration, detailed stress distribution, interference fits, other residual stresses, fretting, flaws, etc. Until the influences of each of these parameters are also represented in the analytical treatment, a sophisticated and precise cumulative damage theory is probably not the greatest need of designers.

Actually, the efficiency of testing is probably improved by conducting program-load tests for the following reasons:

1. Every test provides direct information while several constant amplitude tests must be performed to provide at least a portion of an S-N curve.
2. Variable-amplitude tests are more likely to identify the probable location of failure.
3. Interpretation of results is less dependent upon analysis by a cumulative damage theory, particularly if the loads have been anticipated correctly.

This is not to say that research on cumulative damage is fruitless. A reasonably adequate cumulative damage rule is sorely needed in order to anticipate the way in which fatigue life may be affected by modifications in speed, gross-weight or mission without extensive retesting. (In military aircraft, such changes have modified the frequency of occurrence of a given load level per flight by two orders of magnitude.) Thus research on the problem must be continued vigorously, keeping in mind that the ultimate method must account for practical complexities in the loading sequence. However, procedures for accounting for the quality of the construction are also needed. In many cases ad hoc testing will be required to prove a given design.

Even if all the parameters controlling quality and cumulative damage could be handled analytically, the safety of a given aircraft in a fleet might still be jeopardized by failures initiated by inadvertent flaws or accidental damage. To insure against such failures, fail-safe construction and diligent inspection programs are and will remain necessary. The discussion of fail-safe design procedures is, of course, beyond the scope of this discussion.

#### RECOMMENDATIONS FOR AGARD ACTION

From the foregoing review and the attached bibliography, it is quite clear that the current trends in research on cumulative damage are likely to lead to significant contributions. In contrast to common practice in earlier studies, the practical complexities of both structure and load experience are receiving the proper emphasis. The national aeronautical research centers represented on the Fatigue Committee are taking leading roles in this effort and are generally agreed on the kinds of studies needed. The efforts of these groups complement each other in a rather remarkable way.

Thus, it is difficult to imagine that any new effort organized by AGARD on an international scale is either needed desperately or that it could be expected to lead to new breakthroughs. There is also considerable doubt that a new effort in this area is needed for mutual defense purposes.

One suggestion that might be helpful is that a close liaison be maintained among the research groups working in the various countries. Personal contacts are, of course, most desirable, and several exchanges of visits have taken place during the past several years. With international travel becoming more common, it is reasonable to expect that such visits will be more frequent in the future. AGARD might well encourage these mutual exchanges.

Dr. Schijve offered two specific proposals for possible consideration by the Fatigue Committee:

1. Compile a detailed review of procedures used by aircraft firms to prove the fatigue airworthiness of their designs.

2. Prepare periodic, critical reviews of current investigations on aeronautical fatigue to acquaint designers and other research workers with current advances on cumulative damage testing, random-load fatigue and the effects of residual stresses.

Expertly done, such reviews could be very useful.

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Anon.: Einfluss des Hartverchromens auf die Zeit- und Betriebsfestigkeit eines niedriglegierten Vergutungsstahles mit einer Zugfestigkeit von 200 kp/mm<sup>2</sup>  
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Anon.: Betriebsfestigkeit von Augenstaben aus einer Aluminiumlegierung (Luftfahrtwerkstoff 3.1245.7). LBF Rpt. TM 14/65